12. Use the golden search procedure to find the minimum point to the function in Exercise 1 \( f(x) = 2x^2 - e^x \) accurate to ±0.001. How many iterations are required? Can you know this in advance? How many function evaluations are required?

27. Find the minimum point of the function of Exercise 22 \( f(x, y) = x^2 + 3y^2 - xy + 3x + 2 \) by steepest descent from (0, 0). Terminate the search when the (x, y) values are within (0.001, 0.001) of the true minimum.

30. The function of Exercise 22 \( f(x, y) = x^2 + 3y^2 - xy + 3x + 2 \) is a quadratic function. That means that the minimum point can be found in two tries with the conjugate gradient method. Confirm that this is true when the starting point is (0,0). Then repeat this for starting points at the other corners of the square.

35. Some books illustrate searching methods for Rosenbrock’s function (see Exercise 33) starting from (-1.2, 1). If steepest descent is used from this starting point, in what direction does one move and to what point before changing direction? What is the direction of the second movement? Superimpose these two movements on the graph of level curves that made in Exercise 33.

37. Apply Newton’s method to find the minimum for the function in Exercise 33 starting from (0, 0).

33. A function that is frequently used to illustrate searching methods for a minimum is Rosenbrock’s function:

\[ f(x, y) = 100(y - x^2)^2 + (1 - x)^2 \]
Because both terms are never negative, it is clear by inspection that the minimum is zero and occurs at (1, 1). Graph level curves for $f = 1, 2, 5, 15$, and 25 and observe that the “valley” is narrow and curved.