8. (25%) Find the solution to \( \frac{dy}{dt} = y^2 + t^2 \), \( y(1) = 0 \), at \( t = 2 \) by the Euler method, using \( h = 0.1 \). Repeat with \( h = 0.05 \). From the two results, estimate the accuracy of the second computation. (Hints: use Eq. 5.19 to estimate the error)

11. (25%) Repeat Exercise 8 but use the midpoint method. Are the results the same? If not, which is more accurate?

47. (25%) For the third-order equation
\[
y^{'''} + t y' - 2 y = t, \quad y(0) = y'(0) = 0, \quad y''(0) = 1,
\]
(a) Solve for \( y(0.2), \quad y(0.4), \quad y(0.6) \) by RKF (you can use ode45() in Matlab to run RKF, but you need to list the system of ODE equations and explain how you solve this equation. Please send you code to TA).
(b) Advance the solution to \( t = 1.0 \) with the Adams-Moulton method.
(c) Estimate the accuracy of \( y(1.0) \) in part (b).

60. (25%) Given the boundary-value problem:
\[
\frac{d^2 y}{d\theta^2} + \frac{y}{4} = 0, \quad y(0) = 0, \quad y(\pi) = 2,
\]
which has the solution \( y = 2 \sin \left( \frac{\theta}{2} \right) \)
(a) Solve, using finite difference approximations to the derivative with
\( h = \frac{\pi}{4} \) and tabulate the errors.
(b) Solve again by finite differences but with a value of \( h \) small enough to reduce the maximum error to 0.5\%.
(c) Solve again by the shooting method. Find how large \( h \) can be to have maximum error of 0.5\%. (Please use the secant method to find \( y'(0) \))
66. (Extra credits: 25%) The most general form of boundary condition normally encountered in second-order boundary-value problems is a linear combination of the function and its derivatives at both ends of the region. Solve through finite difference approximations with four subintervals:

\[ x'' - tx' + t^2 x = t^3, \]
\[ x(0) + x'(0) - x(1) + x'(1) = 3, \]
\[ x(0) - x'(0) + x(1) - x'(1) = 2. \]