Image-based Modeling and Rendering

7. Basic data compression

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Data Compression

- More image samples → usually more fidelity.
  - How to keep more samples in the same devices (memory, disks …) ?

- Data compression in multimedia courses.
  - Lossless compression, e.g. Huffman coding
  - Lossy compression, e.g. vector quantization
    - JPEG
    - MPEG
    - MP3 (MPEG 1 Audio layer III)
    - ……

Outline

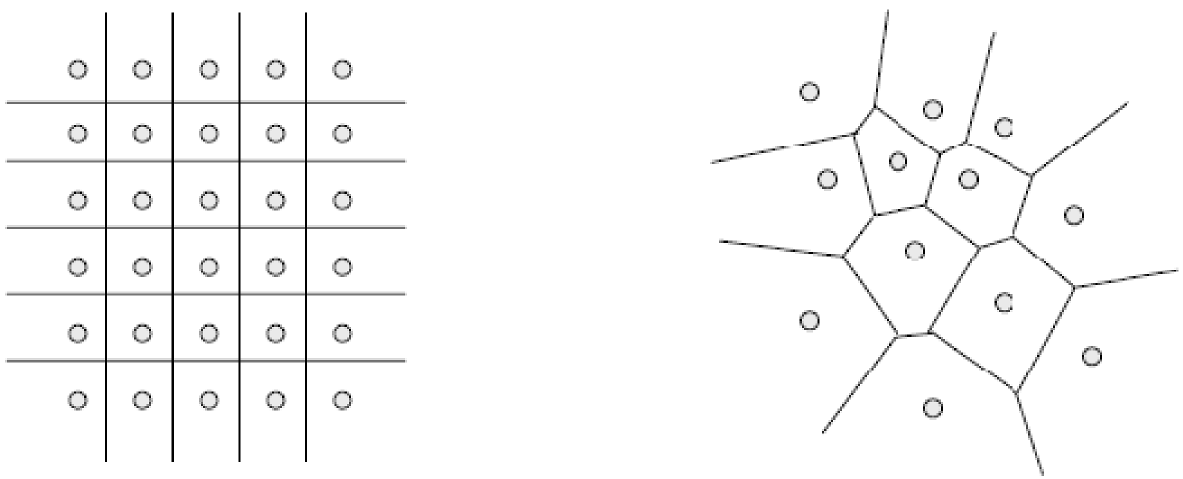
- Vector Quantization (VQ)

- Principal component analysis (PCA)

Vector Quantization (VQ)

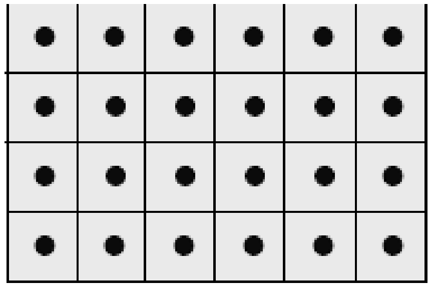
- To project a continuous input space on a discrete output space, while minimizing the loss of information.

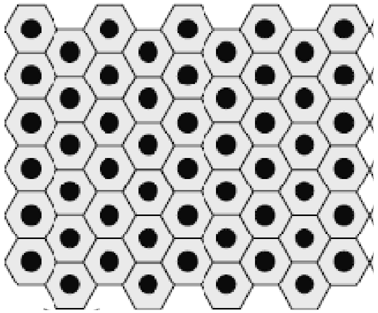
E.g. 2D space



Vector Quantization (cont.)

- VQ =
  - A codebook (set of centroid or codeword, etc.)
  - A quantization function
- E.g.



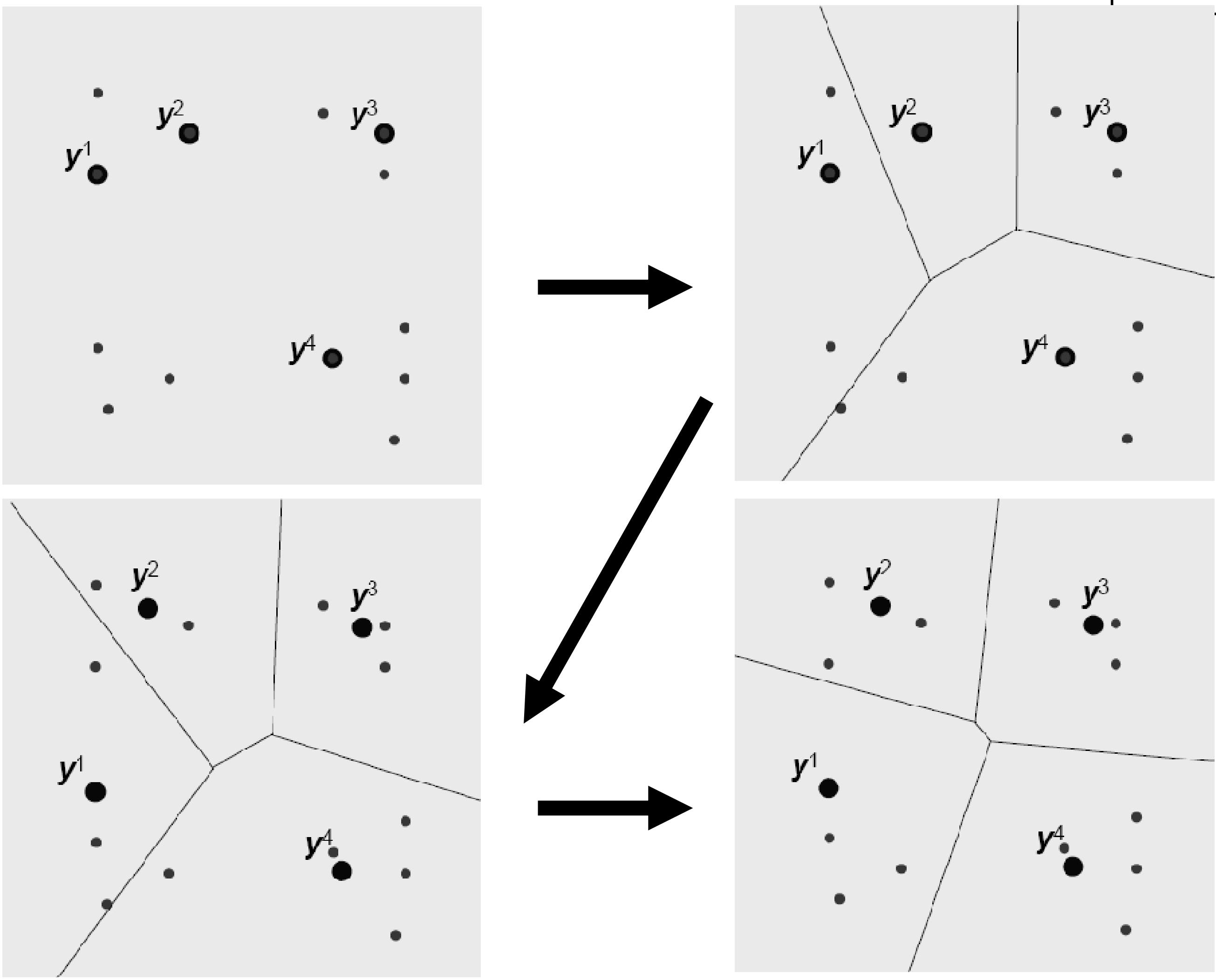


$E_{vq} = 0.962 E_{sq}$

Lloyd's algorithm

1. Choice of an initial codebook.
2. All points x_i are encoded; E_{VQ} is evaluated.
3. If E_{VQ} is small enough, then stop.
4. All centroids y_j are replaced by the center-of-gravity of the data x_i associated to y_j in step 2.
5. Back to step 2.

Lloyd's algorithm



www.dice.ucl.ac.be/~verleyse/lectures/elec2870

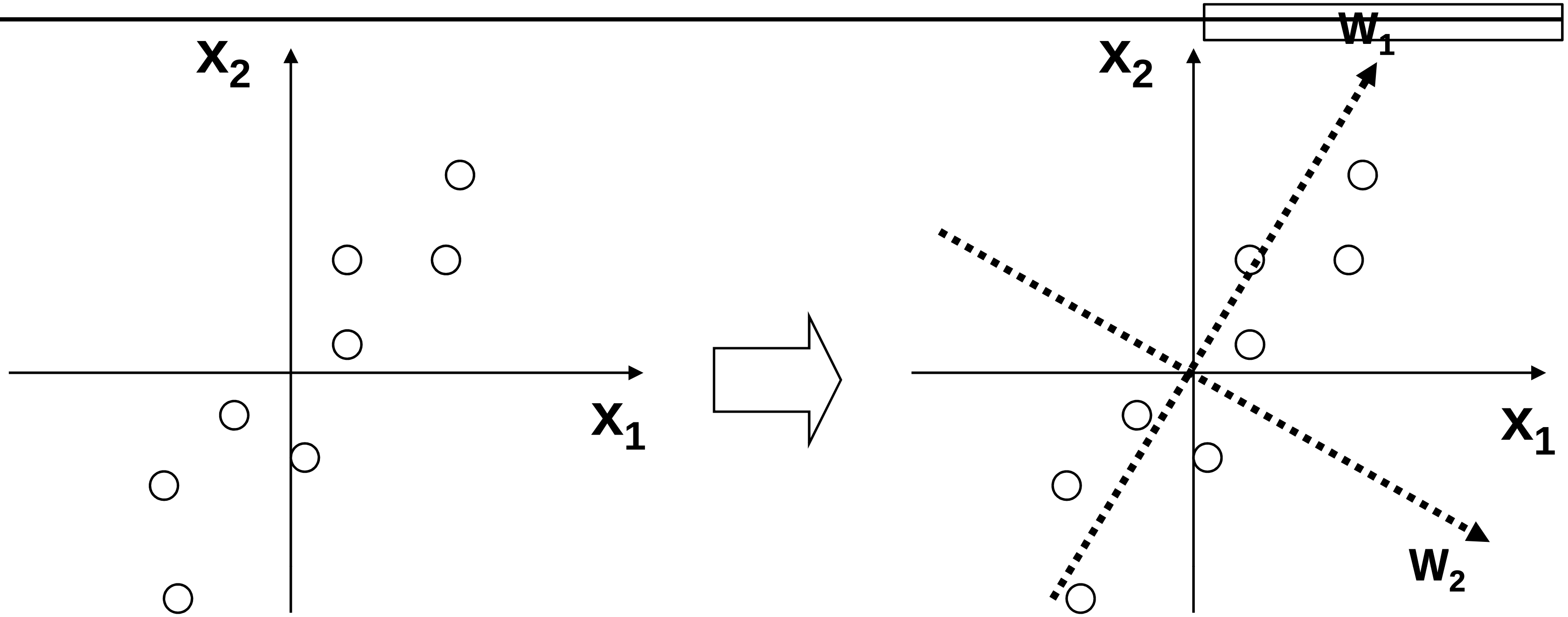
Principal Component Analysis

- Principal component analysis (PCA) is a technique for compression and classification of data.

- Reducing the dimensionality of a data set (sample).
  - by finding a new set of variables, smaller than the original set of variables.
  - Retaining most of a sample's information.

- The new variables, called principal components (PCs), are uncorrelated, and are ordered by the fraction of the total information each retains

PCA



X_1 axis : $(1, 0)^t$

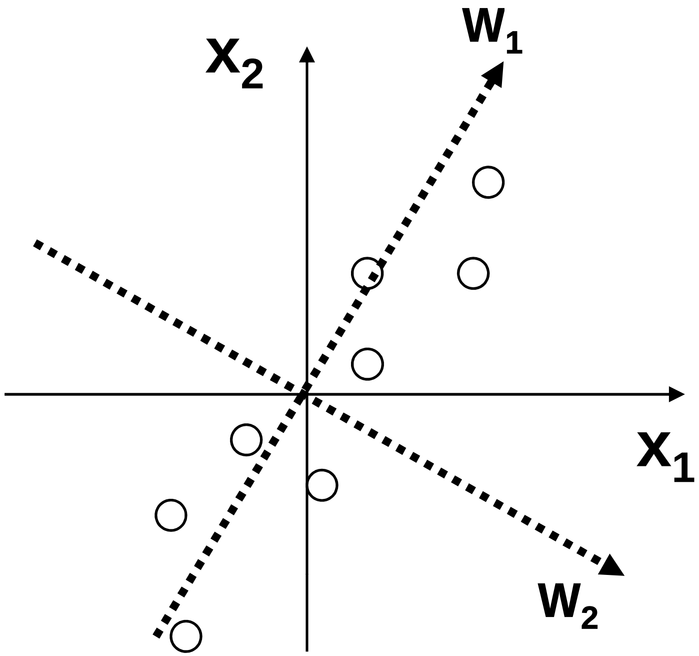
X_2 axis : $(0, 1)^t$

W_1 axis : $(0.5, 0.866)^t$

W_2 axis : $(0,866, -0.5)^t$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 2 \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 2.2321 \cdot \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix} + (-0.134) \cdot \begin{bmatrix} \sqrt{3}/2 \\ -1/2 \end{bmatrix}$$

Principal Components



W_1 axis : $(0.5, 0.866)^t$

W_2 axis : $(0,866, -0.5)^t$

$$2.2321 \cdot \begin{bmatrix} 1/2 \\ \sqrt{3}/2 \end{bmatrix} + (-0.134) \cdot \begin{bmatrix} \sqrt{3}/2 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 1.116 \\ 1.933 \end{bmatrix} + \begin{bmatrix} -0.116 \\ 0.067 \end{bmatrix}$$

Projection on W_1 — Projection on W_2

- The 1st PC W_1 is a minimum ? in X space
- The 2nd PC W_2 is a minimum ? in the plane perpendicular to the 1st PC

Principal Components (cont.)

- PCs are a series of linear least squares fits to samples
  - each PC is orthogonal to all the previous.

- Given a data set of zero mean:

$$w_1 = \arg\max_{\mathrm{w}} \; \mathrm{E}\left((w^T x)^2\right), \text{ where } |w| = 1$$

Max var(z_1)

$$\hat{x}_{k-1} = x - \sum_{i=1}^{k-1} (w_i^T x) w_i$$

Residual of the first k-1 components

$$w_k = \arg\max_{\mathrm{w}} \; \mathrm{E}\left((w^T \hat{x}_{k-1})^2\right), \text{ where } |w| = 1$$

PCA (cont.)

Given a sample of *n* observations on a vector of *p* variables

$$X = (x_1,\ x_2,\ \ldots\ldots x_p)$$

The 1st principal component

$$z_1 = w_1^T X = \sum_{i=1}^{p} w_{1i} x_i$$

w_1 is chosen such that var[z_1] is maximum (for all samples)

$$w_1^T w_1 = 1$$

PCA (cont.)

Likewise, define the k^{th} PC of the sample by

$$z_1 = w_k^T X = \sum_{i=1}^{p} w_{ki} x_i$$

w_k is chosen such that var[z_k] is maximum

subject to $\text{cov}[z_k, z_l] = 0, \text{ for } k > l \geq 1$

and to

$$w_1^T w_1 = 1$$

PCA (cont.)

$$\mathrm{var}[z_1] = E\left(z_1^2\right) - \left(E\left(z_1\right)\right)^2$$

$$= \sum_{j=1}^{p}\sum_{i=1}^{p} w_{1j} w_{1i} E\left(x_i x_j\right) - \sum_{j=1}^{p}\sum_{i=1}^{p} w_{1j} w_{1i} E\left(x_i\right) E\left(x_j\right)$$

$$= \sum_{j=1}^{p}\sum_{i=1}^{p} w_{1j} w_{1i} S_{ij}$$

$$= w_1^T S w_1$$

S is the covariance matrix for the samples

PCA (cont.)

To find w_1 that maximize var[z_1] , subject to $|w_1|=1$

Let λ be a Lagrange multiplier

then maximize $$w_1^T S w_1 - \lambda\left(w^T w - 1\right)$$

the differentiation should be 0 $$S w_1 - \lambda w_1 = 0$$

Therefore, w_1 is an eigenvector of S

PCA (cont.)

Since we want to maximize

$$\mathrm{var}[z_1] = w_1^T S w_1$$
$$= w_1^T \lambda_1 w_1 = \lambda_1 w_1^T w_1 = \lambda_1$$

So λ_1 is the largest eigenvalue of S

w_1 is the correspondent eigenvector

PCA (cont.)

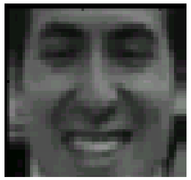
- From similar deduction, K^{th} PC is the eigenvector corresponding to the K^{th} largest eigenvalue.
- The k^{th} largest eigenvalue of S is the variance of the k^{th} PC.
- The k^{th} PC retains the kth greatest fraction of the variation in the sample.

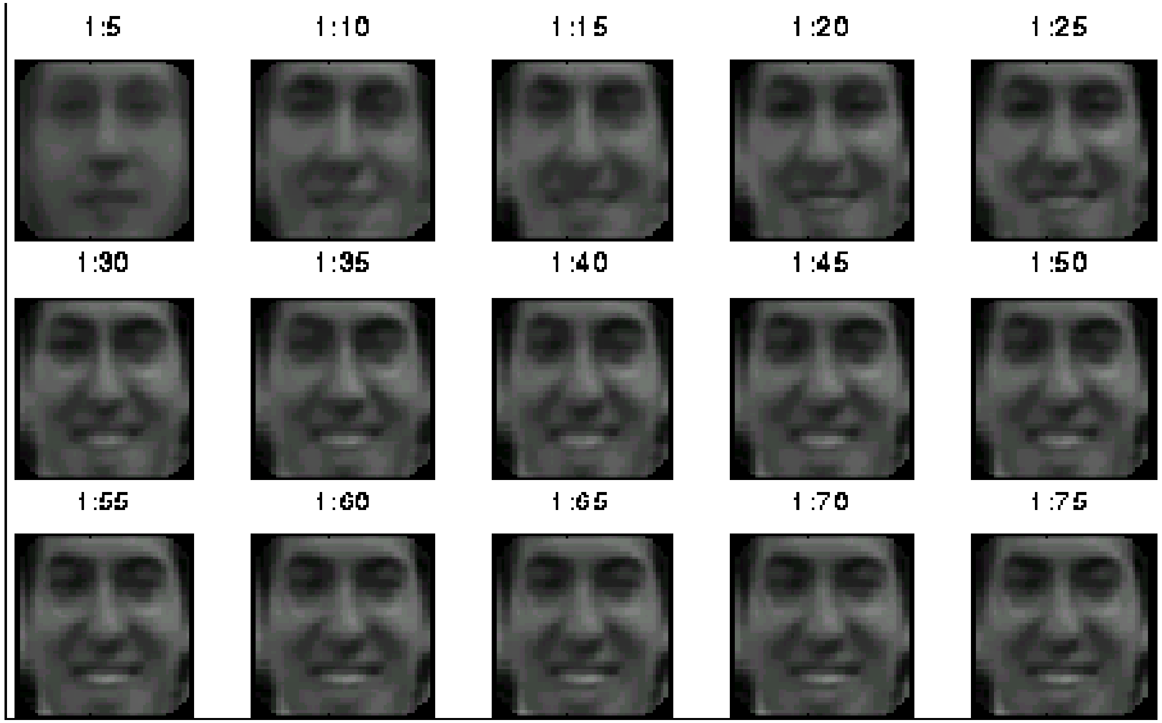
Calculating PCA

1. Calculate the mean.
2. Subtract the mean.
3. Calculate the covariance matrix S.
4. Calculate the eigenvalues and eigenvectors S.
5. Choose the components.
6. Derive the new data set.

Applications of PCA

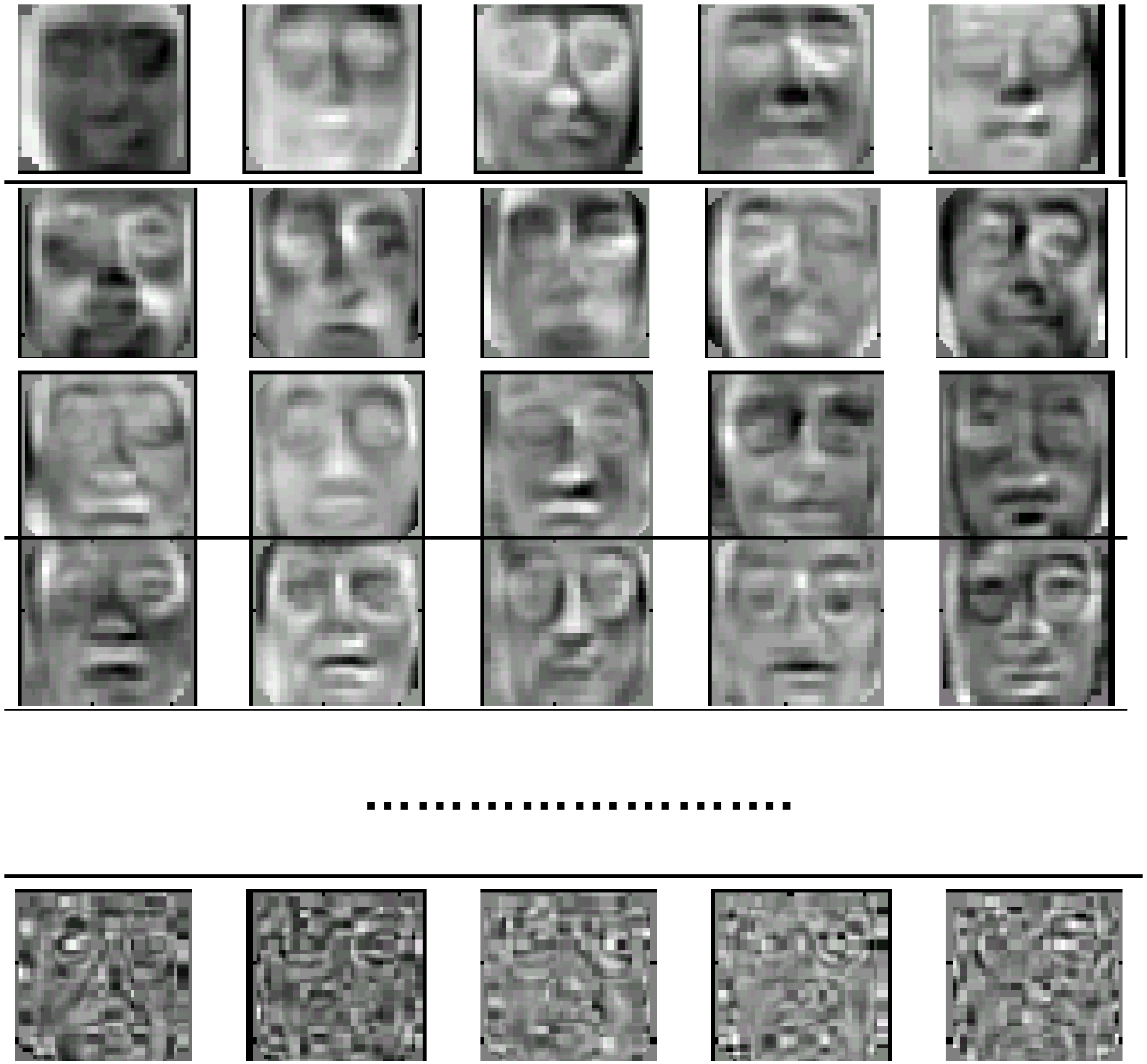
- Eigenfaces





http://www.stat.ucla.edu/~dinov/

Applications of PCA



Principal
Components

Applications of PCA

- Data compression
  - Keep "important" information
- Data analysis
  - Reduce dimensions for classification or recognition.
- There're efficient algorithms of PCA (in memory and computation)

Applications of PCA

Input images

Initialization

3D reconstruction with texture

V. Blanz, T. Vetter, “A Morphable Model For The Synthesis Of 3D Faces”, Proc. SIGGRAPH’99, pp. 187-194.