Human Computer Interaction

6. Smart and suggestive interfaces (C)

Data Scattering and RBF

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Introduction

- Given a set of samples, what are the in-between values ?

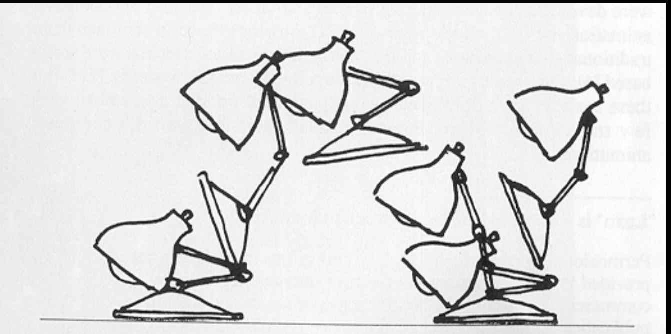


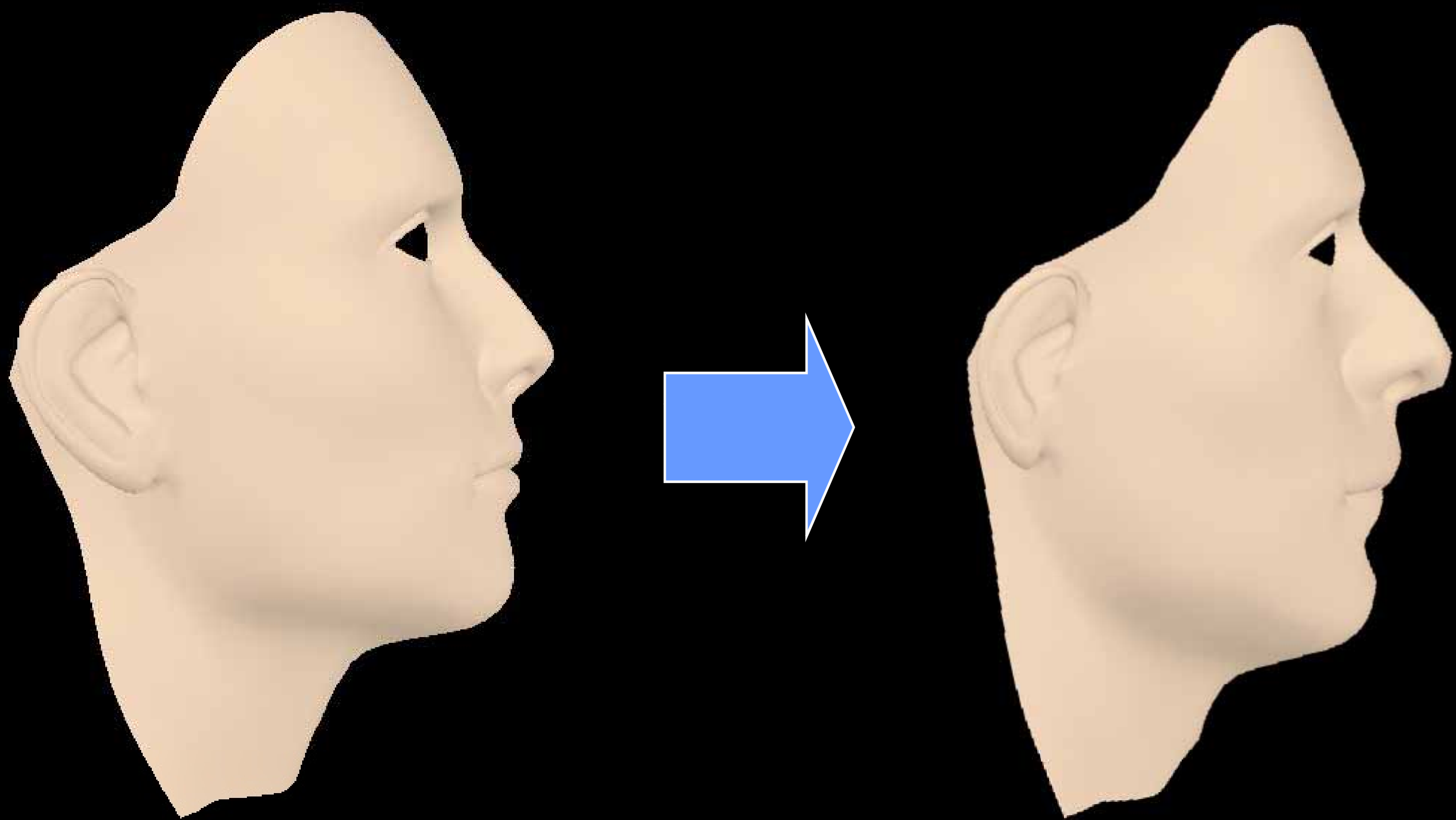
FIGURE 3. Squash & stretch in Luxo Jr.'s hop.

- Linear interpolation, interpolating by splines, …
  - It works for structured data.

How about unstructured or scattered data samples?

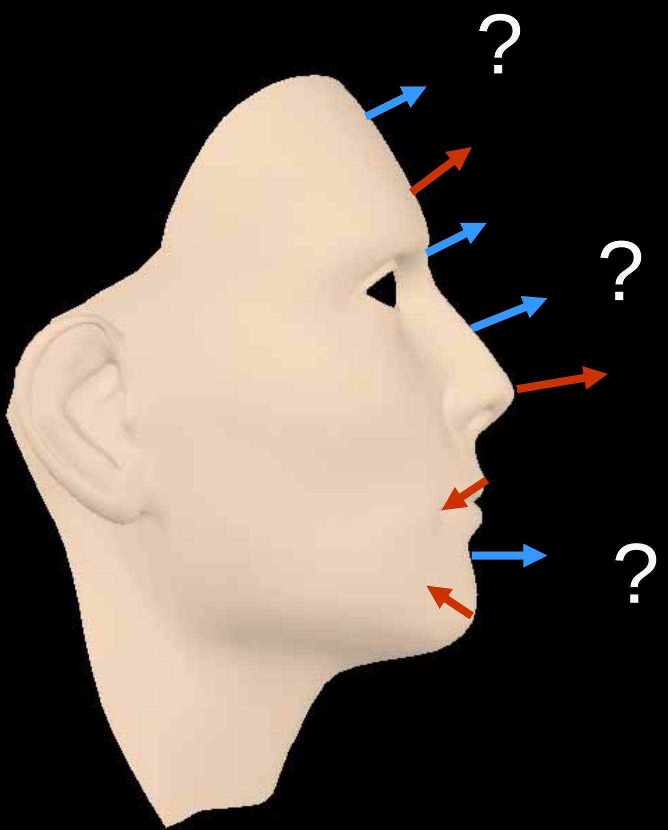
Scattered Data Interpolation

For instance, head model adjustment…



Scattered Data Interpolation

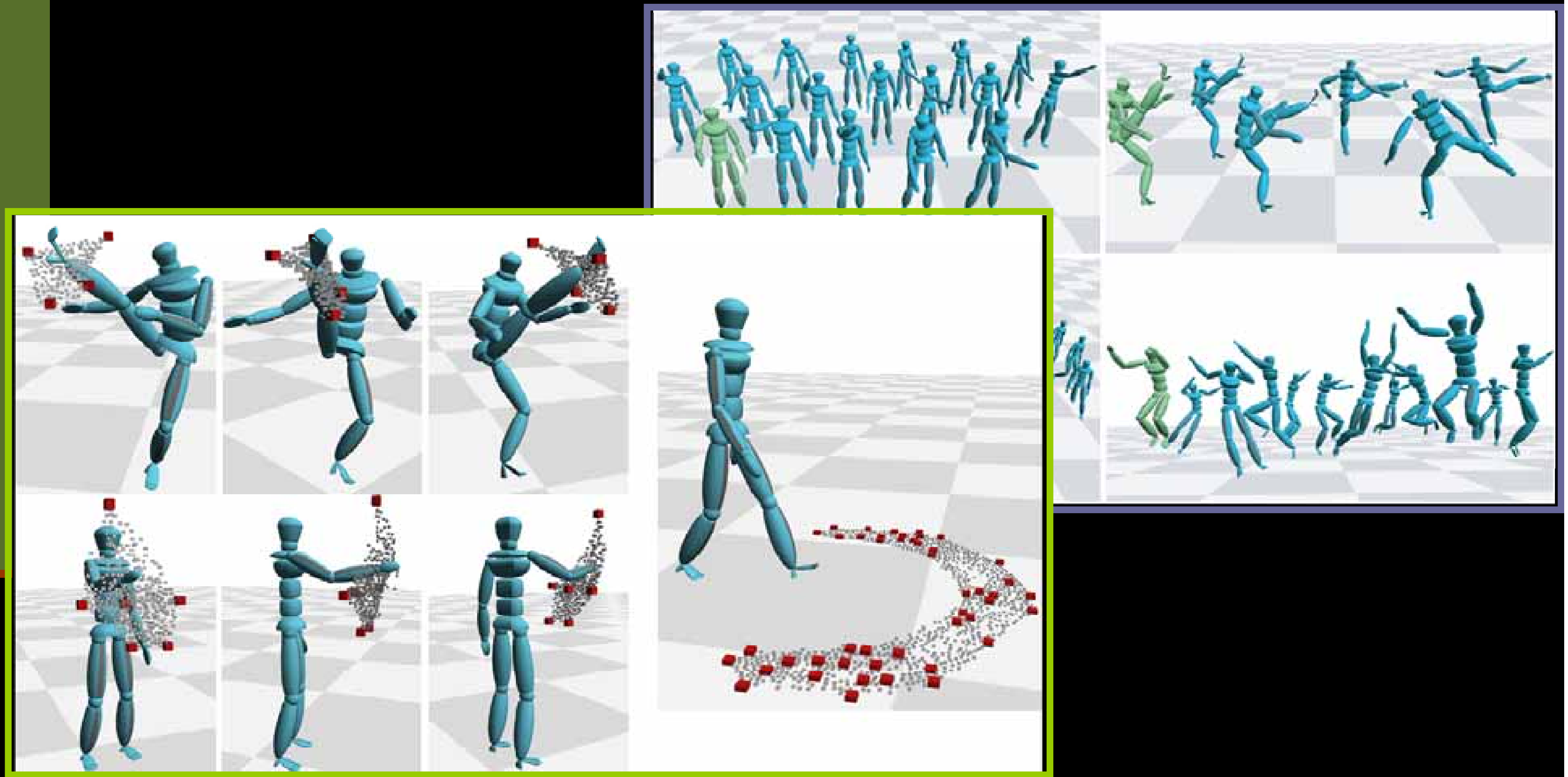
You can drag all vertices (more than 6000) or drag feature samples...



Drag by users

What are the displacement ?

Scattered Samples



Kover et al, “Automated Extraction and Parameterization of Motions in Large Data Sets”, Proc. SIGGRAPH’04.

Interpolation

- Characteristics
  - Interpolation vs. Extrapolation
  - Linear Interpolation vs. Higher Order
  - Structured vs. Scattered
  - 1-Dimensional vs. Multi-Dimensional
- Techniques
  - Splines (cubic, B-splines, ...)
  - Series (polynomial, radial basis functions, ...)
  - ....
  - Exact solution, minimization, fitting, approximation

Scattered Data Interpolation

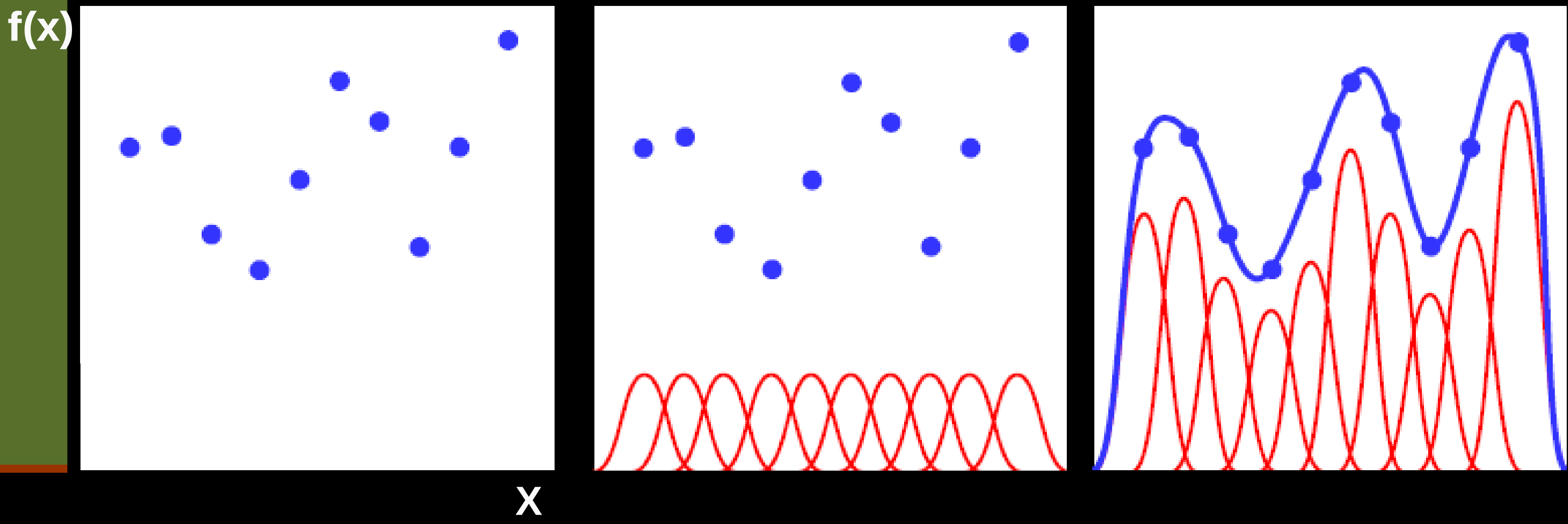
- Given N samples ($\mathbf{x}_i$, f_i), such that $S(\mathbf{x}_i)=f_i$, We would like to reconstruct a function $S(x)$.

- Actually, there're infinite solutions.

- Reasonable constraints:
  - $S(\mathbf{x})$ should be continuous over the entire domain
  - We want a 'smooth' surface

- Radial basis functions are popularly used solutions.

Radial Basis Function



R.Gutierrez-Osuna, Intro to Pattern Analysis, Texas, A&M Univ.

The RBF Solution

$$S(\mathbf{x}) = \sum_{i=1}^{N} \lambda_i \phi\left(\left\|\mathbf{x} - \mathbf{x}_i\right\|\right) + M(\mathbf{x})$$

- λ_i is the *weight* of centre $\mathbf{x}_i$
- $\phi(\mathrm{r})$ is the *basic function*
- $M(\mathbf{x})$ is a low-degree polynomial
- $\| \; \|$ is the Euclidean norm

Ref: http://www.unknownroad.com/rtfm/rbf_rms2002.ppt

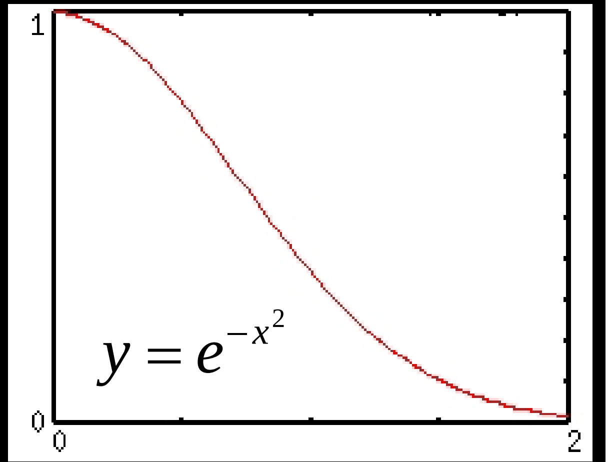
Radial Basis Functions

- ϕ can be any function
  - For instance,

$$\phi(r) = r^3$$

$$\phi(r) = e^{-kr^2}$$

$$\phi(r) = \sqrt{r^2 + k^2}$$



- M(x) = a + bx + cy + dz
  - 4 coefficients a,b,c,d
  - Adds 4 equations and 4 variables to the linear system

Finding an RBF Solution

- The weights and polynomial coefficients are unknowns
- We know N values of f_j:

$$s(\mathbf{x}_j) = \sum_{i=0}^{N} \lambda_i \phi\left(\left\|\mathbf{x}_j - \mathbf{x}_i\right\|\right) + M(\mathbf{x}_j)$$

- N equations $f_j = S(x_j)$

$$f_j = \lambda_1 \phi\left(\left\|\mathbf{x}_j - \mathbf{x}_1\right\|\right) + \ldots + \lambda_N \phi\left(\left\|\mathbf{x}_j - \mathbf{x}_N\right\|\right) + a + bx_j + cy_j + dz_j$$

- 4 additional constraints

$$\sum_{i=1}^{N} \lambda_i = \sum_{i=1}^{N} \lambda_i x_i = \sum_{i=1}^{N} \lambda_i y_i = \sum_{i=1}^{N} \lambda_i z_i = 0$$

The Linear System Ax = b

$$\sum \lambda_i \phi_{ji} + P(x_j) = f_i$$

$$\sum \lambda_i = 0$$

$$\sum \lambda_i x_i = 0$$

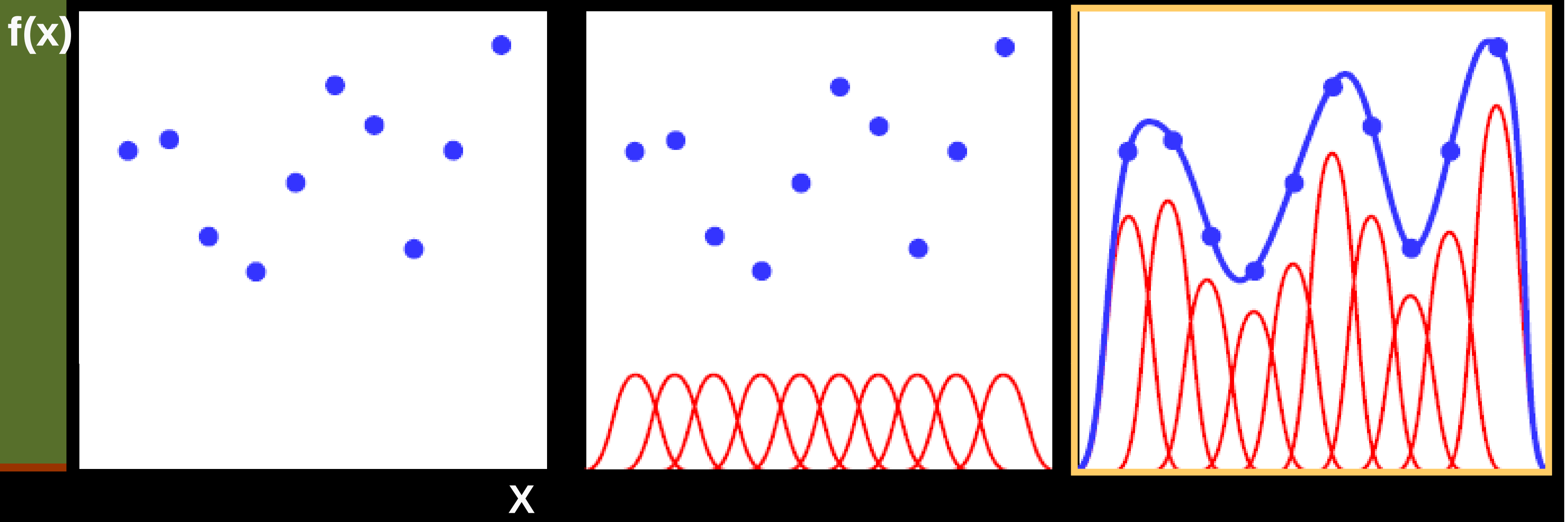
$$\sum \lambda_i y_i = 0$$

$$\sum \lambda_i z_i = 0$$

$$\begin{bmatrix} \phi_{11} & \cdots & \phi_{1N} & 1 & x_1 & y_1 & z_1 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{N1} & \cdots & \phi_{NN} & 1 & x_N & y_N & z_N \\ 1 & \cdots & 1 & 0 & 0 & 0 & 0 \\ x_1 & \cdots & x_N & 0 & 0 & 0 & 0 \\ y_1 & \cdots & y_N & 0 & 0 & 0 & 0 \\ z_1 & \cdots & z_N & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_N \\ a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} f_1 \\ \vdots \\ f_N \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $\phi_{ji} = \phi\left(\left\|\mathbf{x}_j - \mathbf{x}_i\right\|\right)$

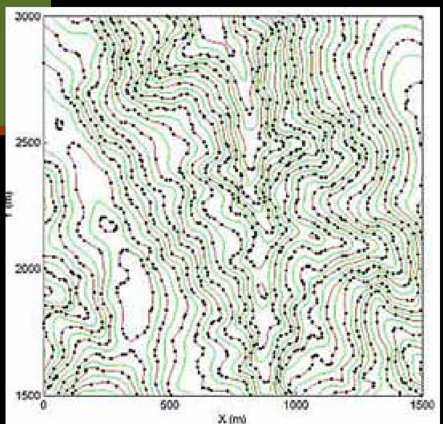
Radial Basis Function

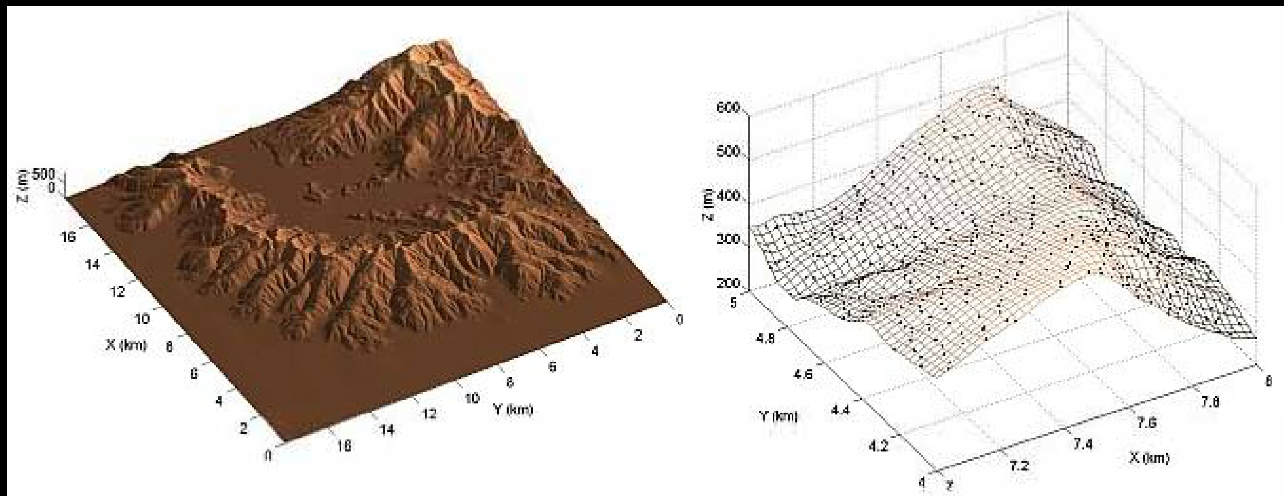


- λ_i define the influence of the centre
- After constructing S(x), the interpolation or extrapolation can be easily performed.

Applications of RBF

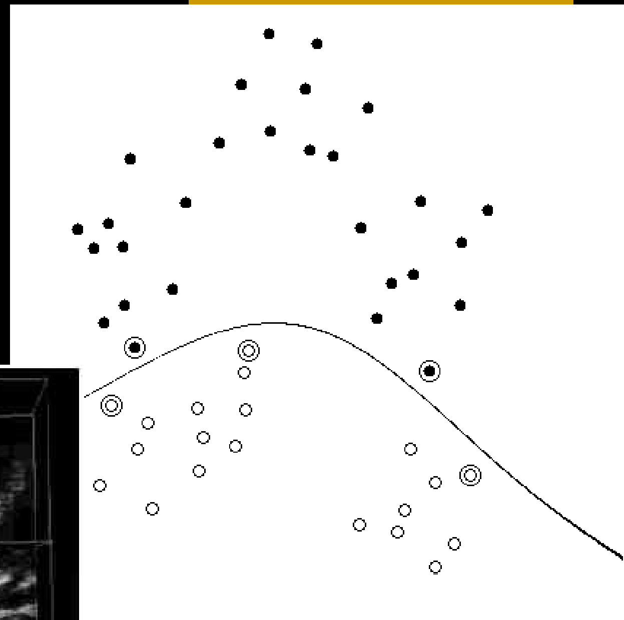
- Image/Object warping, morphing
- Surface reconstruction
- Range scanning, geographic surveys, medical data
- Field Visualization (2D and 3D)
- AI





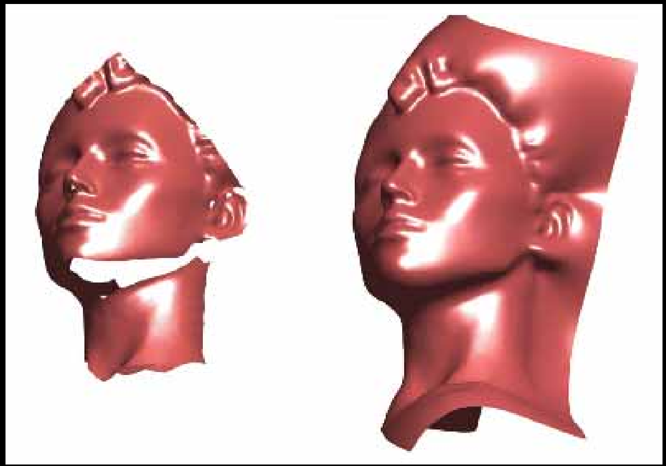
Applications of RBF

http://www.kyb.tuebingen.mpg.de/bu/people/bs/svm.html

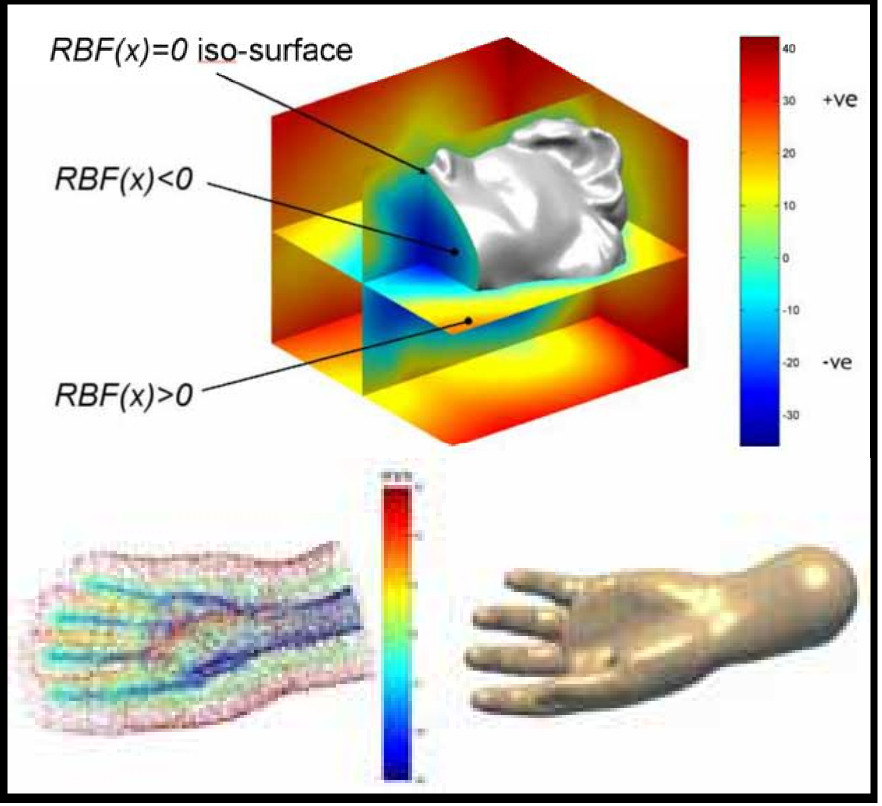


http://mrcas.mpe.ntu.edu.sg/research/urobot/registration.htm

Applications of RBF in Graphics



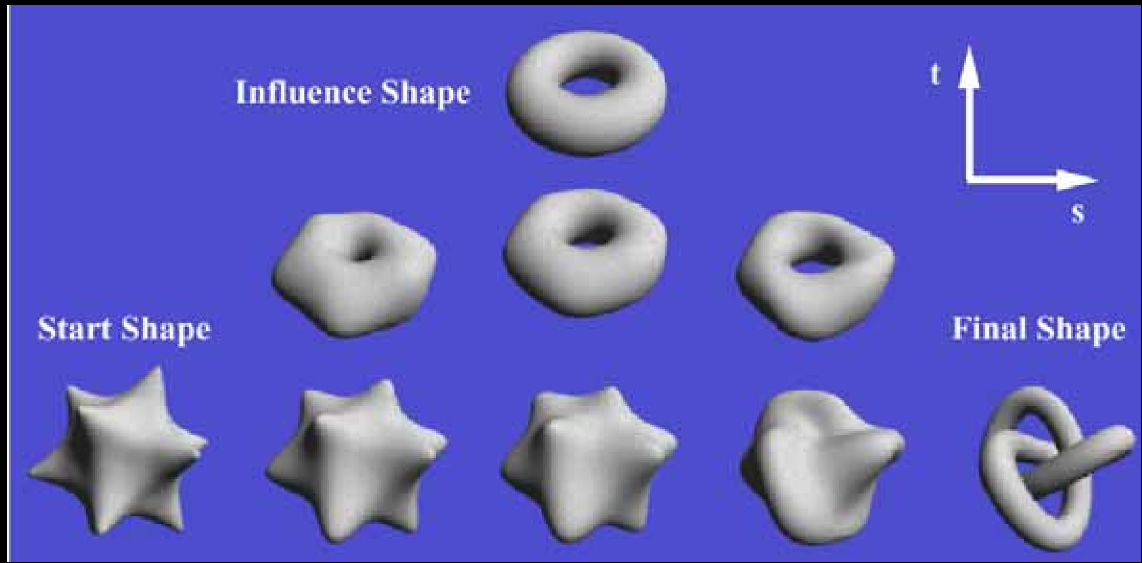
Hole filling



Surface reconstruction

http://www.farfieldtechnology.com/products/toolbox/theory/surfacefaq.html

Applications of RBF in Graphics



Morphing with influence shapes

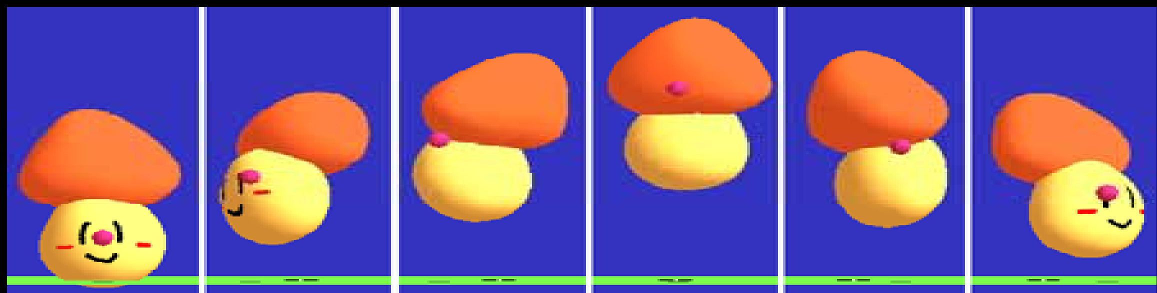
G. Turk and J. O'Brien, "Shape Transformation Using Variational Implicit Functions," Proc. SIGGRAPH'99.

Keyframing with RBF

- In traditional temporal keyframing, key poses are defined at specific points in time.
- How to manipulate key poses with an intuitive interface?
- T.Igarashi, et al. “Spatial Keyframing for Performance-driven Animation”, Proc. SCA’05.

Animation Authoring with RBF

Define key poses

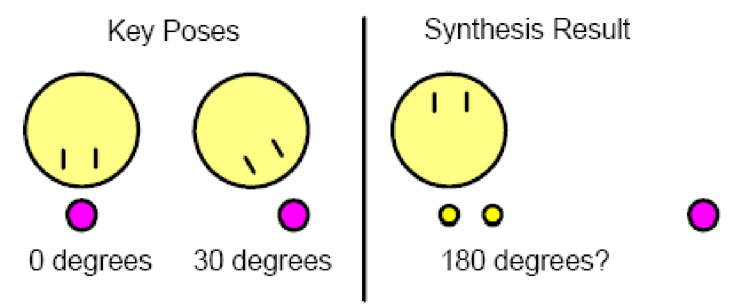


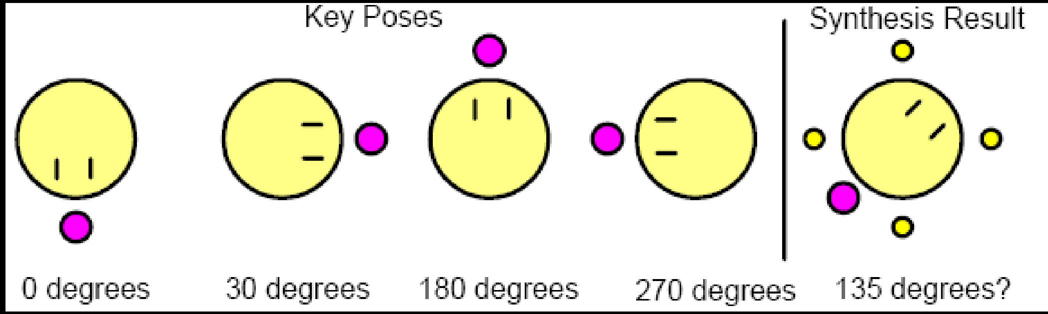
Spatially keyframing animation

Animation Authoring with RBF

- Using RBF
  - X: the position of control cursors.
  - S: pose, a set of local transformations of the body parts.

- Why not angular parameters
  - Since interpolating in 3D space not in sequential time space.
  - And …





Animation Authoring with RBF