

# Human Computer Interaction

## 6. Fundamental of Vision-based Interfaces (B)

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# Goals

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- From 2D to 3D, learn the basic 3D reconstruction.
- Efficient (and simple ?) approaches for real-time user interfaces.

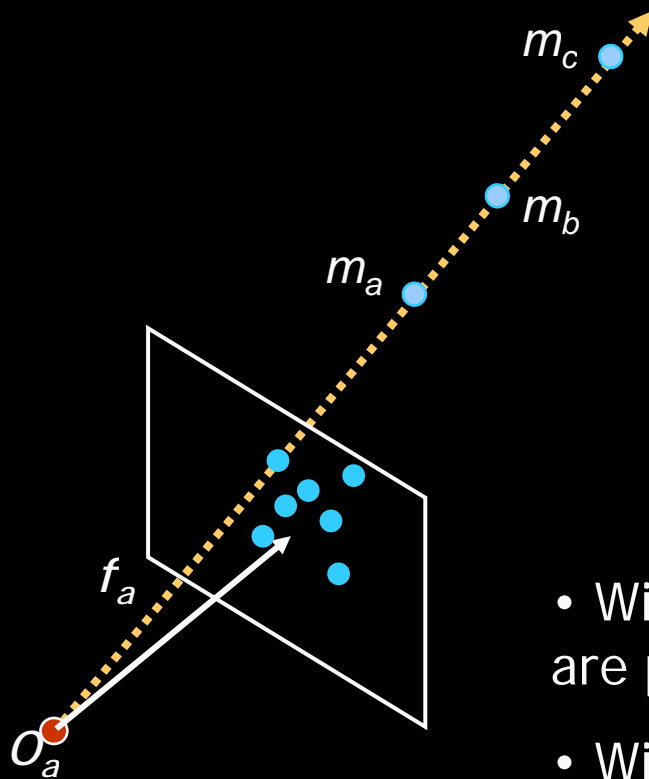
# Outline

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- Feature extraction
  - Color matching
  - Grouping or clustering
  - Silhouette & foreground
- Motion tracking
  - Filtering and prediction
- 3D position estimation
  - Two views
  - mirrored views

# Estimating 3D positions

- Given a projected point set, what are the 3D structure?

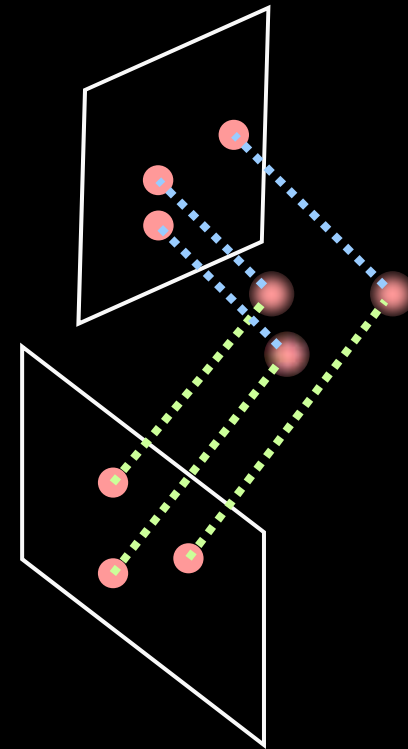


Which one is correct?

- Without other information, all these points are possible!!!
- With prior constraints or **at least two views** !

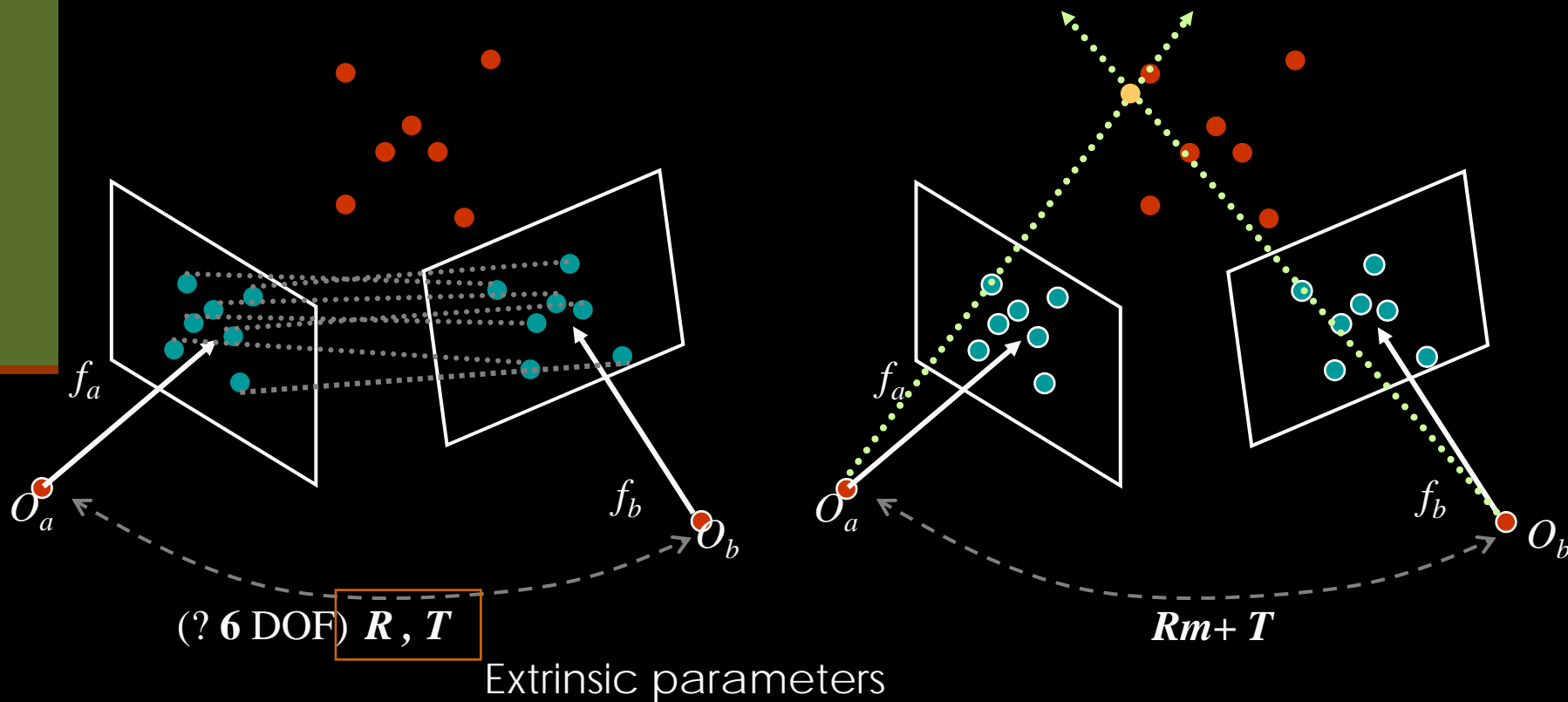
# Two Orthogonal Views

- The simplest case:
  - two orthogonal views + parallel projection
- Orthogonal views
  - Calibration problems ?
- Parallel projection
  - When does it work?
- If the applications do not require high accuracy in 3D est., this can be a candidate approach.



# Triangulation (Stereo)

- Given some points in **correspondence** across two or more images (taken from calibrated cameras),  $\{(u_j, v_j)\}$ , compute the 3D location  $X$



# Triangulation (Stereo)

- Constructing 3D structure from two views.
  - H.C. Longuet-Higgins (Nature'81).
  - J.Weng et al. *The two-view approach* (PAMI'89).
- Given some points in **correspondence** across two images (in a normalized camera model),  $\{(u_j, v_j)\}$ ,
  - Estimate  $R, T$  from corresponding points.
  - 3D position estimation from triangulation.
  - (optional) non-linear optimization

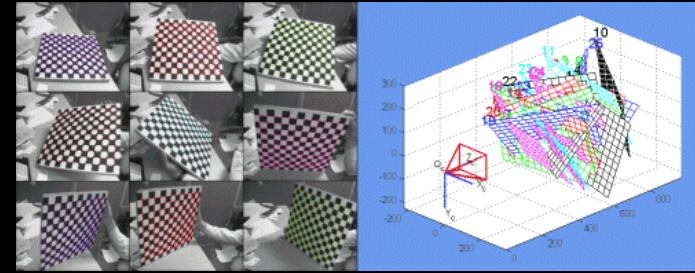
# 3D Estimation from Two Views

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1. Estimate intrinsic camera parameters.
  - E.g. optic center, camera distortion, etc.
2. Estimate extrinsic camera parameters.
  - E.g. motion between two views.
3. Estimate 3D structure by triangulation.
  - Refer to textbooks or lecture notes in computer vision (or image-based modeling) courses.



# Camera Calibration



- Public camera calibration tools
  - A flexible new technique for camera calibration
    - <http://research.microsoft.com/~zhang/calib/>
    - Z. Zhang. A flexible new technique for camera calibration. IEEE Trans. Pattern Analysis and Machine Intelligence, 22(11):1330-1334, 2000.
  - Camera calibration toolbox for matlab
    - [http://www.vision.caltech.edu/bouquetj/calib\\_doc/](http://www.vision.caltech.edu/bouquetj/calib_doc/)
  - Tsai's camera model
    - <http://www.cs.cmu.edu/~rgw/TsaiDesc.html>
    - "A versatile Camera Calibration Technique for High-Accuracy 3D Machine Vision Metrology Using Off-the-Shelf TV Cameras and Lenses", Roger Y. Tsai, IEEE J. Robotics and Automation, Vol. RA-3, No. 4, 1987, pages 323-344.

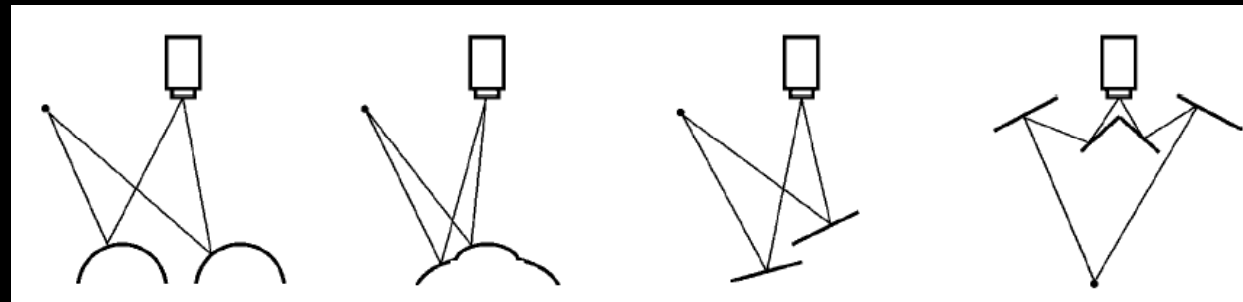
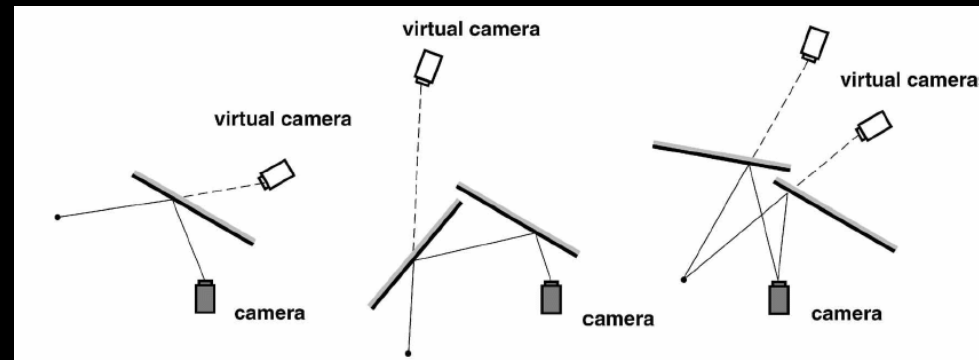
# Triangulation limits

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- Difficult to reliably estimate structure and motion unless:
  - large ( $x$  or  $y$ ) rotation
  - large field of view and depth variation
- Camera calibration is important
- Need good feature trackers or manual assistance
- Post-processing of the resulting 3-D points?

# Estimating 3D Positions

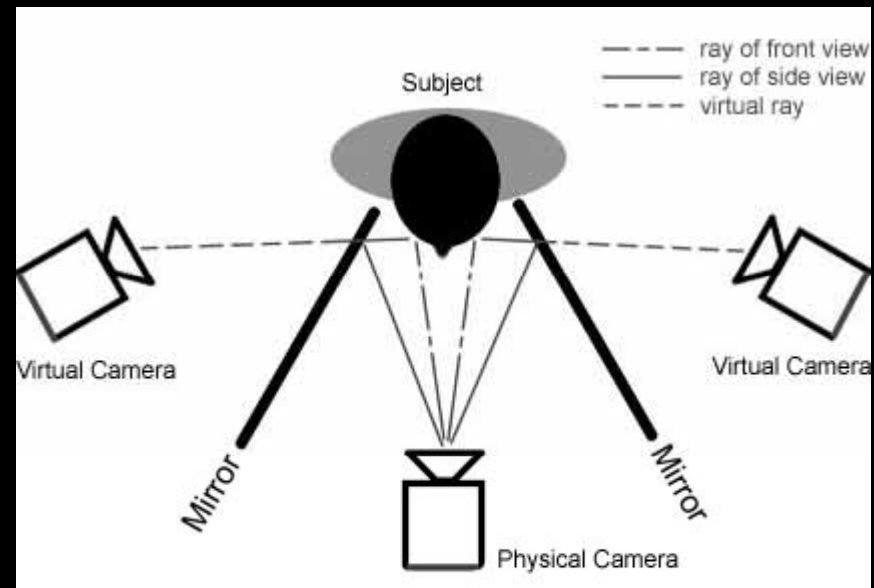
- Using mirrors, we can acquire multi-views with a single camera.



Stereo sensors from a variety of mirrors. J. Gluckman et al. (CVPR'99)  
(PAMI'02)

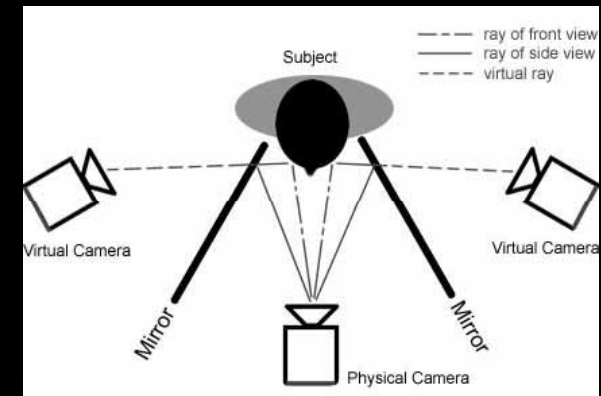
# Mirrored Views

- Using planar mirrors can make reconstruction much easier.
- The simplest case:
  - Mirrors placed at 45-degree included angles.
  - Problem:
    - Calibration
    - Projective views
    - ...



# Reconstruction from Mirrored Views

- E.C. Patterson et al. (CA'91):
  - Assumed that the mirror and camera was vertical.
- S. Basu et al. (CVPR workshop'97)(ICCV'98):
  - Lip position evaluation via  $R, t$  estimation between virtual cameras.



- I.-C Lin et al. (CG&A'02)(TVC' 05)
  - Efficient and reliable algorithms for calibration and 3D reconstruction in planar mirror configuration.
  - For dense facial motion tracking.

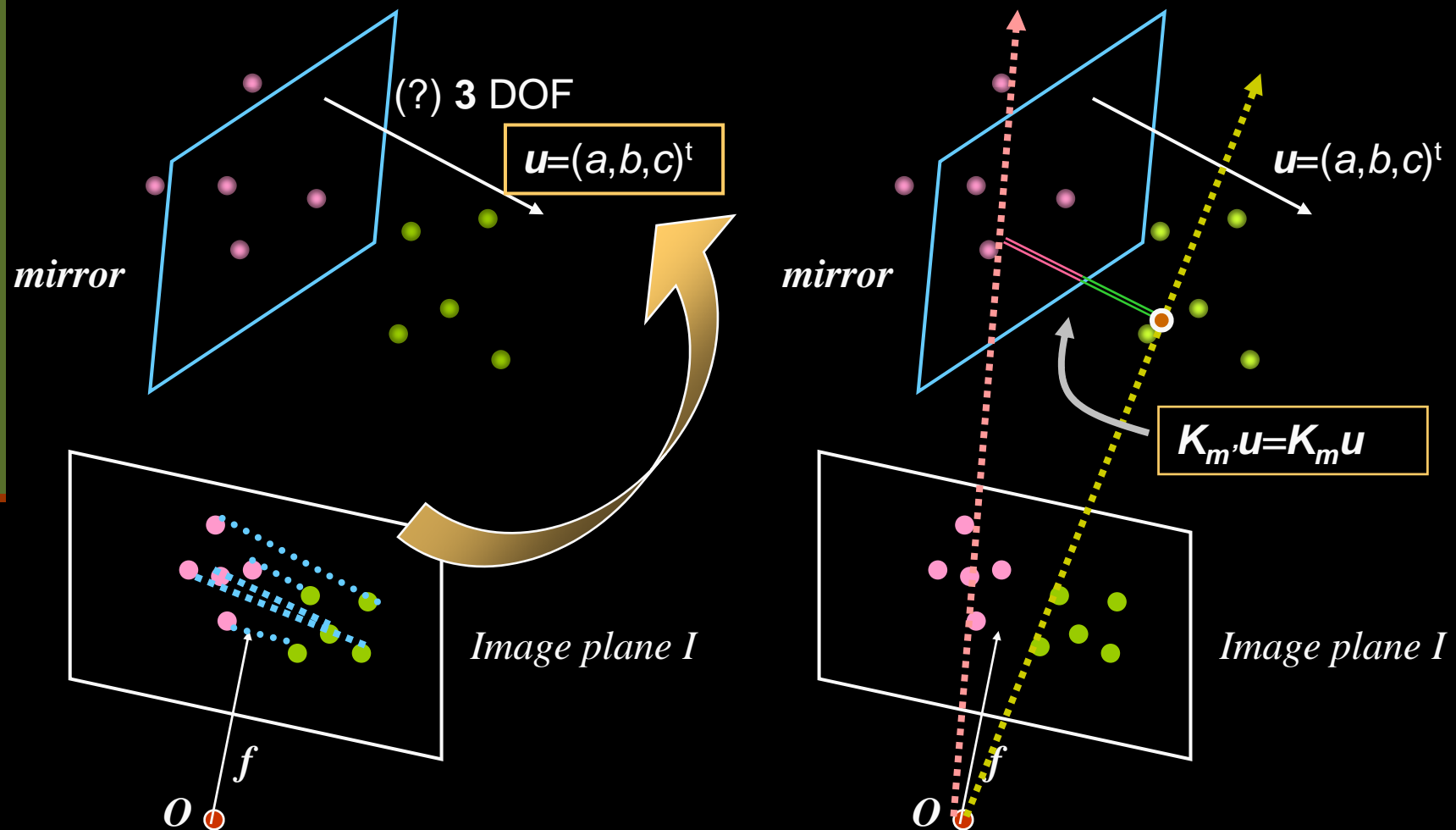
# 3D Tracking in Mirror-reflected Multi-views



“Mirror Mocap” (illuminated by UV light)

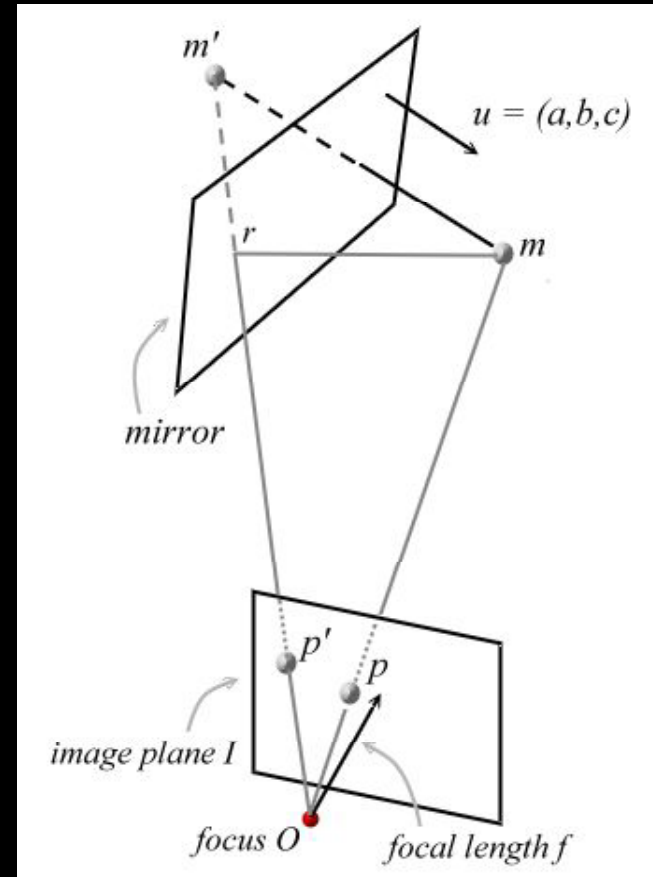
# 3D Position estimation

- Estimating 3D positions by evaluating the mirror plane's parameters.



# 3D Position estimation

- Given real vs. mirrored projected point correspondences.
- Known:  $p_i, p_i', f$ .
- Unknown:  $m_i, m_i', u, d$ .
- Calculating 3D positions via mirror plane estimation.

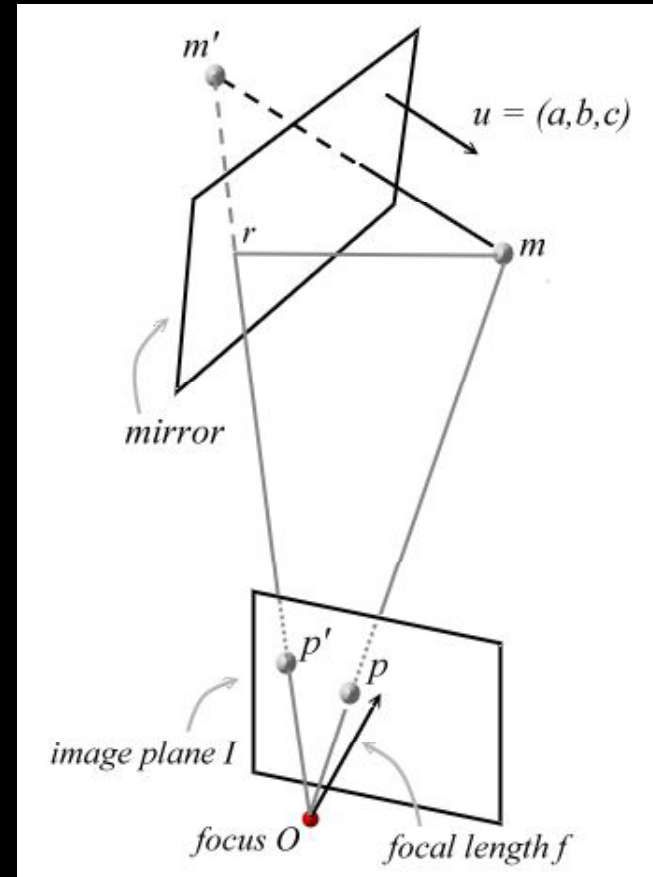


The geometric representation of physical point  $m$ , reflected point  $m'$ , and the projected point  $p$  and  $p'$ .



# 3D Position estimation (cont.)

- We assumed that the mirror is flat.
- Calculating 3D positions via evaluation of the mirror plane.
- Properties:
  - $ax+by+cz=d$ ,  $u=(a, b, c)^t$ ,  
 $|u|=1$ . (1)
  - $m_i'=m_i + ku$ . (2)
  - $(m_i' - \Theta)=H_u(m_j - \Theta)$ , where  $H_u$   
is the Householder matrix. (3)



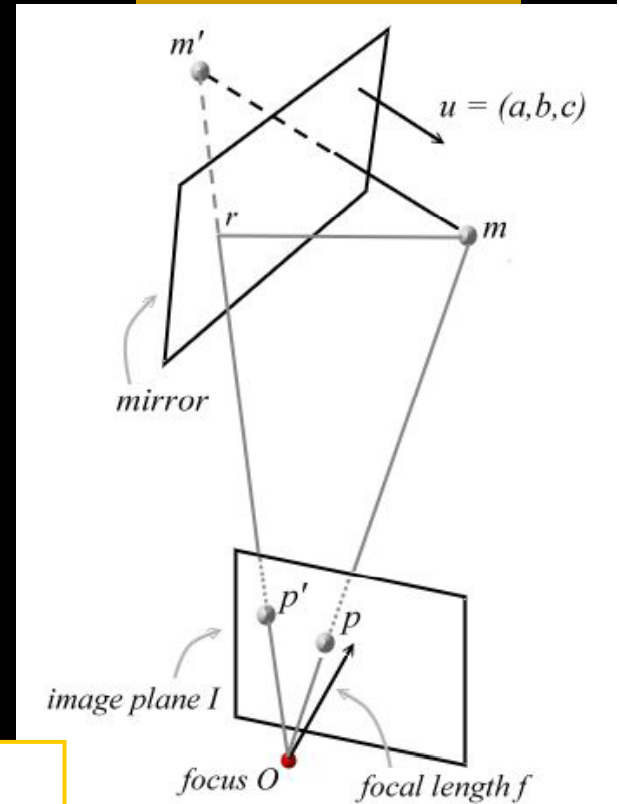
# 3D Position estimation (cont.)

- Since  $m_i, m_i'$  and  $u$  are coplanar,

$$(p'_i)^t U p_i = 0, \text{ where } U = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix}$$

- Approximate the mirror plane by a least square method.

$$\begin{bmatrix} (y_{p1} - y'_{p1})f & (-x_{p1} + x'_{p1})f & (x_{p1}y'_{p1} - y_{p1}x'_{p1}) \\ (y_{p2} - y'_{p2})f & (-x_{p2} + x'_{p2})f & (x_{p2}y'_{p2} - y_{p2}x'_{p2}) \\ \vdots & \vdots & \vdots \\ (y_{pn} - y'_{pn})f & (-x_{pn} + x'_{pn})f & (x_{pn}y'_{pn} - y_{pn}x'_{pn}) \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$



# 3D Position estimation (cont.)

- Deducing from symmetric properties, depths are in proportion to  $d$ . (similar to  $T$  in stereovision)

$$\begin{bmatrix} \left(\frac{2a^2-1}{2f}\right)x_{pi} + \left(\frac{ab}{f}\right)y_{pi} + ac & \frac{x'_{pi}}{2f} \\ \left(\frac{ab}{f}\right)x_{pi} + \left(\frac{2b^2-1}{2f}\right)y_{pi} + bc & \frac{y'_{pi}}{2f} \\ \left(\frac{ac}{f}\right)x_{pi} + \left(\frac{bc}{f}\right)y_{pi} + \frac{2c^2-1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} z_{mi} \\ z'_{mi} \end{bmatrix} = d \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

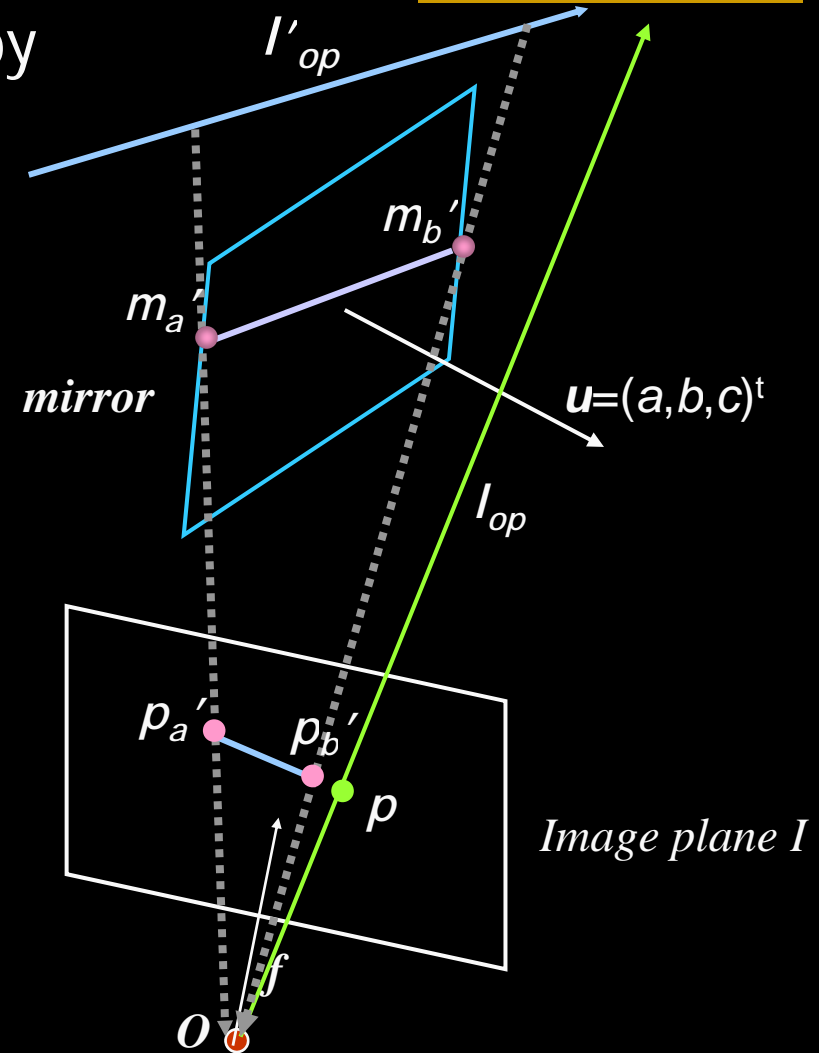
- ( $z_{mi}$  and  $z'_{mi}$ ) can be estimated by a least square method. The 3D positions are reconstructed by scaling data.

# Potential 3D Candidates

- Constructing 3D candidates by *mirrored epipolar lines*.

$$(p')^t U p = 0$$

$$\begin{bmatrix} x'_p & y'_p & 1 \end{bmatrix} \begin{bmatrix} -cy_p + b \\ cx_p - a \\ -bx_p + ay_p \end{bmatrix} = 0$$



# Conclusion

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- Efficient (or simple) 3D reconstruction algorithms are introduced.
- 3D vision-based controls can provide more flexibility.
- The process time and device requirement will increase for 3D.
- Choose an appropriate method according to accuracy, budgets, process time, devices, etc.

# Appendix

- H.C. Longuet-Higgins (Nature'81).
- J.Weng et al. *The two-view approach* (PAMI'89).

# The Two-view Approach

- Without loss of generality, the images of different view direction  $d_1, d_2$  is regarded as a rigid-body motion of an object between  $t_1, t_2$ .

$\mathbf{x}_i = (x_i, y_i, z_i)$  is the 3D position of point  $P_i$  at time  $t_1$ .

$\mathbf{x}_i' = (x_i', y_i', z_i')$  is the 3D position of point  $P_i$  at time  $t_2$ .

$\mathbf{X}_i = (u_i, v_i, 1)$  is the projected vector of  $P_i$  at time  $t_1$ .

$\mathbf{X}_i' = (u_i', v_i', 1) = (x_i'/z_i', y_i'/z_i', 1)$  is the projected vector of  $P_i$  at time  $t_2$ .

# The Two-view Approach (1)

- Step (1). Solving for essential matrix E.

$$A = \begin{bmatrix} u_1 u'_1 & u_1 v'_1 & u_1 & v_1 u'_1 & v_1 v'_1 & v_1 & u'_1 & v'_1 & 1 \\ u_2 u'_2 & u_2 v'_2 & u_2 & v_2 u'_2 & v_2 v'_2 & v_2 & u'_2 & v'_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_n u'_n & u_n v'_n & u_n & v_n u'_n & v_n v'_n & v_n & u'_n & v'_n & 1 \end{bmatrix}$$

- $\min_h || Ah || = 1$ , subject to  $|| h || = 1$ .

$$E = [E_1 \quad E_2 \quad E_3] = \sqrt{2} \begin{bmatrix} h_1 & h_4 & h_7 \\ h_2 & h_5 & h_8 \\ h_3 & h_6 & h_9 \end{bmatrix}$$

The solution of  $h$  is the unit eigenvector of  $A^t A$  associated with the smallest eigenvalue.



# The Two-view Approach (2)

- Step (2). Determining a unit vector  $T_s$  with  $T_0 = \pm T_s$ .
  - $\min_{T_s} || E^t T_s ||$ , subject to  $|| T_s || = 1$ .

The solution of  $T_s$  is the unit eigenvector of  $EE^t$  associated with the smallest eigenvalue.

$$E = T_{\times} R = [E_1 \ E_2 \ E_3] = [T_{\times} R_1 \ T_{\times} R_2 \ T_{\times} R_3]$$
$$\therefore E_1, E_2, E_3 \perp T \Rightarrow E^t T_s = 0$$

- if  $(\sum_i (T_s \times X_i') \cdot (E X_i) < 0)$ ,  $T_s = -T_s$ .

# The Two-view Approach (3)

- Step (3). Determining rotation matrix  $R$ .

- Without noise,  $W=R$

$$W = [(E_1 \times T_s + E_2 \times E_3) \quad (E_2 \times T_s + E_3 \times E_1) \quad (E_3 \times T_s + E_1 \times E_2)]$$

Using the identity equation:  $(a \times b) \times c = (a \cdot c) b - (b \cdot c) a$

- With noise,

$\min_R || R - W ||$ , subject to:  $R$  is a rotation matrix.

# The Two-view Approach (3 app.)

- $\min_R || RC - D ||$ , subject to:  $R$  is a rotation matrix.

$$B = \sum_{i=1}^3 B_i^t B_i$$

- Define a 4x4 matrix  $B$  by

$$B_i = \begin{bmatrix} 0 & (C_i - D_i)^t \\ D_i - C_i & [D_i + C_i]_{\times} \end{bmatrix}_{4 \times 4}$$

where

- $q = (q_0, q_1, q_2, q_3)^t$  is the unit eigenvector of  $B$  associated with the smallest eigenvalue.

$$R = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 + q_0 q_3) & 2(q_1 q_3 - q_0 q_2) \\ 2(q_2 q_1 + q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_3 q_1 - q_0 q_2) & 2(q_3 q_2 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

# The Two-view Approach (4)

- Step (4). Checking  $T = 0$ , If  $T \neq 0$ , determine the sign of  $T_0$ .

if for all  $i = 1 \sim n$ , then report  $T \approx 0$ .

else if  $(\sum_i (T_s \times X_i') \cdot (R X_i) > 0)$ , then  $T_0 = T_s$ ,

otherwise  $T_0 = -T_s$ .

# The Two-view Approach (5)

- Step (5). If  $T \neq 0$ , estimate relative depths.

- To find 
$$Z_i = \left( \frac{z'_i}{\|T\|}, \frac{z_i}{\|T\|} \right)^t = (\tilde{z}'_i, \tilde{z}_i)$$

by 
$$\min \left\| \begin{bmatrix} X'_i & -RX_i \end{bmatrix} Z_i - T^0 \right\|$$

# The Nonlinear Optimization

- Two-view linear algorithms are often easily disturbed by noise.
  - More calibration points.
  - Nonlinear optimization.
- First, take the result of the two-view linear algorithm as an initial guess.
- Approximate the  $R, T$  by  $\min_m \{ || f(u, m) || \}$  in a nonlinear least square approach
  - E.g. the Levenberg-Marquardt method, or the Gauss-Newton method.
  - $f(u, m) = prj(m, y(u, m)) - u$

where  $u$  is the observed projected position,  
 $m$  is the motion parameters( $R, T$ ),  $y(u, m)$  is the best 3D positions of  $P$ ,  
and  $prj(m, x)$  is the projected position of the input structure  $x$  and motion  $m$ .